Generation of Equivalent Circuit from Finite Element Model Using Model Order Reduction

Yuki Sato[1] and Hajime Igarashi[1], *Member IEEE*

[1]Graduate School of Information Science and Technology, Hokkaido University, Sapporo, 060-0184, Japan,
yukisato@em-si.eng.hokudai.ac.jp

This paper presents a novel generation method of equivalent circuits for FE model of electromagnetic devices using model order reduction (MOR) based on Padé approximation via the Lanczos process (PVL). In this method, an equivalent circuit is directly generated from the reduced transfer function which is obtained using MOR based on PVL algorithm. It is shown that the generated circuit yields sufficiently accurate results in both frequency and time domains. Moreover, the computational time required for the present method is much shorter than identification of circuit parameters by frequency sweep based on the original FE models.

***Index Terms*— Equivalent circuits, finite element analysis, reduced order systems.**

I. Introduction

FINITE ELEMENT METHOD (FEM) has been widely used to design electric machines and electromagnetic devices. On the other hand, because of heavy computational burden in FE analysis, electromagnetic devices have often been modeled by the equivalent circuit method for design of their control and driving circuits. The lumped parameters in equivalent circuit can be obtained from loss and reactance computed by FE analysis. However, it is difficult using this method to express the frequency characteristics of electromagnetic apparatus at high accuracy. The lumped parameters can also be determined directly from the frequency response [1]. The computational time required for FE computations at different frequencies would be, however, too expensive for real uses.

Recently it has been pointed out that the equivalent circuit can directly be generated from the exact solutions to eddy current problems [2]. Although this method is promising from aspects of numerical accuracy and computational complexity, it can only treat simple structures such as plate and cylinder.

In this paper, the authors propose a new method to generate equivalent method directly from FE models using model order reduction (MOR). In this method, the equivalent circuit is generated from the reduced transfer function of the original system obtained by Padé approximation via the Lanczos (PVL) process [3][4]. This method can be applied to complicated objects and it provides highly accurate results over all frequency range with small computational burden. It is also possible to employ MOR based on the proper orthogonal decomposition for this purpose. We will discuss this method in another paper [5].

II. Formulation

A. Padé via Lanczos

Let us consider the FE equation of quasi-static electromagnetic field coupled with electric circuits of the form

$$\mathrm{N}\dot{\boldsymbol{x}} + \mathrm{K}\boldsymbol{x} = \boldsymbol{b}v \tag{1a}$$

$$i = \boldsymbol{l}^t \boldsymbol{x} \tag{1b}$$

where N, $\mathrm{K}\in\mathrm{R}^{n\times n}$, $\boldsymbol{b}$, $\boldsymbol{l}$, $\boldsymbol{x}\in\mathrm{R}^n$, v and i are voltage and current, respectively, where n is number of degree of freedom. The unknowns vector $\boldsymbol{x}$ is composed of vector and scalar potentials and current. The transfer function of this system can be expressed by

$$\mathrm{H}(s) = \boldsymbol{l}^t(\mathrm{K} + s\mathrm{N})^{-1}\boldsymbol{b} \tag{2}$$

Moreover, H(s) is expressed around the expansion point s_0 as

$$\mathrm{H}(s_0+\sigma) = \boldsymbol{l}^t(\mathrm{I} - \sigma\mathrm{A})^{-1}\boldsymbol{r} \tag{3}$$

where $\mathrm{A}=-(\mathrm{K}+s_0\mathrm{N})^{-1}\mathrm{N}$ and $\boldsymbol{r}=(\mathrm{K}+s_0\mathrm{N})^{-1}\boldsymbol{b}$. We apply spectral decomposition to A in (3) to obtain

$$\mathrm{H}(s_0+\sigma) = \boldsymbol{l}^t(\mathrm{I} - \sigma\mathrm{S}\Lambda\mathrm{S}^{-1})^{-1}\boldsymbol{r} = \sum_{i=1}^{n}\frac{f_i g_i}{1-\sigma\lambda_i} \tag{4}$$

where Λ and S are matrices including the eigenvalues and eigenvectors of A, $\boldsymbol{f}=\mathrm{S}^t\boldsymbol{l}$ and $\boldsymbol{g}=\mathrm{S}^{-1}\boldsymbol{b}$, respectively. It is possible to obtain the equivalent circuit from (4). However, this formulation would be unsuitable for real uses because of heavy computational burden in solution of the eigenvalue problem.

To circumvent the above difficulty, we use Lanczos method [3] in which we consider the tridiagonal matrix whose eigenvalues correspond to the significant eigenvalues of A. That is, using the Lanczos method, we obtain the reduced transfer function

$$\mathrm{H}_q(s_0 + \sigma) = \boldsymbol{l}^t\boldsymbol{r}\boldsymbol{e}_1^t(\mathrm{I} - \sigma\mathrm{T}_q)^{-1}\boldsymbol{e}_1 \tag{5}$$

where $\boldsymbol{e}_1=[1,0,\ldots,0]^t$ and T_q is the $q\times q$ tridiagonal matrix where q is much smaller than n. Note here that computation of the eigenvalues and eigenvectors of T_q is much lighter than that of A. Applying the spectral decomposition to T_q in (5), we obtain the reduced transfer function

$$\mathrm{H}_q(s_0 + \sigma) = \boldsymbol{l}^t\boldsymbol{r}\boldsymbol{e}_1^t(\mathrm{I} - \sigma\mathrm{S}_q\Lambda_q\mathrm{S}_q^{-1})^{-1}\boldsymbol{e}_1 = \sum_{j=1}^{q}\frac{\boldsymbol{l}^t\boldsymbol{r}\mu_j\nu_j}{1-\sigma\lambda_j} \tag{6}$$

where Λ_q and S_q are matrices including eigenvalues and eigenvectors of T_q, $\boldsymbol{\mu}=\mathrm{S}_q^{\,t}\boldsymbol{e}_1$ and $\boldsymbol{\nu}=\mathrm{S}_q^{-1}\boldsymbol{e}_1$, respectively. Expression (6) corresponds to the Padé approximation of the transfer function (3).

B. Generation of equivalent circuit

Now the reduced transfer function (6) is regarded as an admittance function Y_q as the input and output of (1) are set to voltage and current. We redefine (6) for simplicity as

$$Y_q(s_0+\sigma) = k_\infty + \sum_{\substack{j=1 \\ \lambda_j \neq 0}}^{q} \frac{k_j}{\sigma - p_j} \tag{7}$$

where

$$k_j = \frac{-\boldsymbol{l}^t \mathbf{r} \mu_j \nu_j}{\lambda_j}, \quad k_\infty = \sum_{\substack{j=0 \\ \lambda_j = 0}}^{q} \boldsymbol{l}^t \mathbf{r} \mu_j \nu_j, \qquad p_j = \frac{1}{\lambda_j} \tag{8}$$

The expansion point is set to $s_0=2\pi f_{\max}$ assuming that frequency range of interest is $0 \leq f \leq f_{\max}$. Substituting $\sigma=-2\pi f_{\max}+\mathrm{j}\omega$ into (7), we can obtain

$$\begin{aligned} Y_q(s_0+\sigma) &= k_\infty + \sum_{j=1}^{q} \frac{k_j}{-2\pi f_{\max} + \mathrm{j}\omega - p_j} \\ &= \frac{1}{R_\infty + \mathrm{j}\omega L_\infty} + \sum_{j=1}^{q} \frac{1}{R_j + \mathrm{j}\omega L_j} \end{aligned} \tag{9}$$

From (9), it is easy to generate the Foster equivalent circuit shown in Fig. 1 under the condition of $|p_j|>2\pi f_{\max}$.

III. Results

To test the validity of this method, we apply this method to the inductor model with AC voltage source and external lumped circuit shown in Fig. 2 in which μ_r and κ are set to 10 and 5×10^6S/m, respectively. Moreover, the circuit parameters v, R and L are 1V, 10^{-5} Ω and 10^{-10} H, respectively. We analyze the model in both frequency and time domains. The frequency range of interest is $0 \leq f \leq 1000$Hz. In the time domain analysis, the driving frequency is set to 300Hz.

The circuit currents for the frequency and time domain analyses are shown in Figs. 4 and 5 in which "FEM" and "$q=i$" indicate the results obtained by FE analysis and the present method, in which i is number of stages of the Foster circuit. From Figs. 4 and 5, it is found that we can reduce the differences between the currents obtained by present method and FE analysis by increasing the number of the stages in both frequency and time domains. The computational time to obtain the results shown in Fig. 4 using present method is about 27% of that in FE computations at 30 different frequencies. This method would be very effective for design of external circuit connected to FE models of electromagnetic machines and devices because of its light computational burden and accuracy.

In the long version, we will discuss generation of Cauer circuit. Moreover, we will discuss validity of this method applied to modeling of antennas and moving objects.

This work was supported in part by JSPS KAKENHI Grant Number 25630101, as well as JSPS and CAPES under the Japan-Brazil Research Cooperative Program.

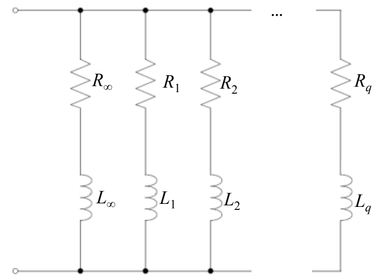

Fig. 1 Foster Circuit

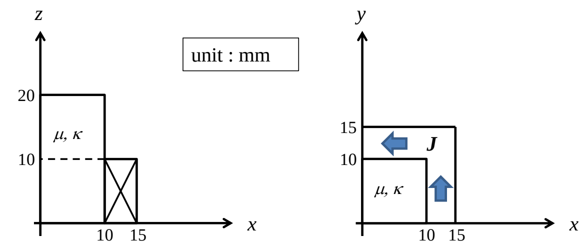

Fig. 2 Inductor model

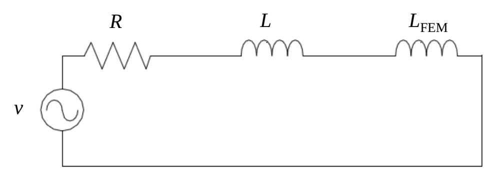

Fig. 3 Circuit including FE model

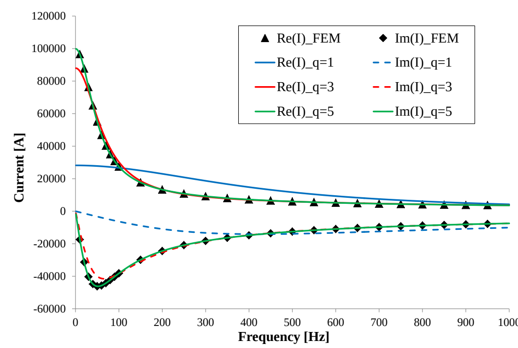

Fig. 4 Frequency chacteristics obtained by present method and FEM.

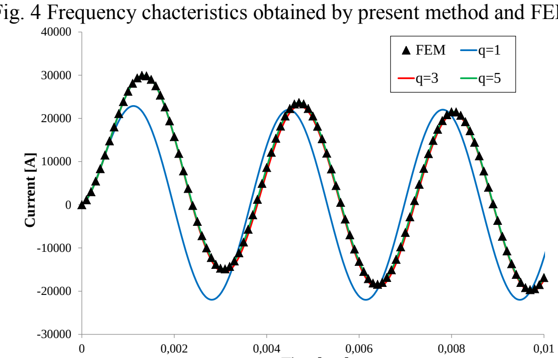

Fig. 5 Transient currents obtained by present method and FEM.

References

[1] B. Gustavsen, A. Semlyen, "Rational Approximation of Frequency Domain Responses by Vector Fitting," *IEEE Trans. Power Delivery*, vol. 14, no. 3, pp. 1052-1061, 1999.

[2] Y. Shindo, O. Noro, "Simple Circuit Simulation Models for Eddy Current in Magnetic and Wires," *IEEJ Trans. Fundamentals and Materials*, vol. 134, no. 4, pp. 173-181, 2014.

[3] P. Feldmann and R. A. Freund, "Efficient Linear Circuit Analysis by Padé Approximation via the Lanczos Process," *IEEE Trans. Computer-Aided Design*, vol. 14, no. 5, pp. 639–649, May 1995.

[4] M. Celik, A. C. Cangellaris, "Simulation of Dispersive Multiconductor Transmission Lines by Padé Approximation via the Lanczos Process," *IEEE Trans. Microwave Theory and Techniques,* vol. 1996.

[5] T. Shimotani, Y. Sato and H.Igarashi, "Generation of Electric Circuit from Finite Element Model Using Proper Orthogonal Decomposition," *submitted to Abstract of Compumag2015*.